A Parametric Analysis of the Effect of a Material Supply Line Delay in Stock Management[1]

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Abstract
It is known that the presence of a material supply line delay may lead to unwanted oscillatory stock behavior. It is also well known that fully considering the supply line in the ordering decisions, which means using the same adjustment time for stock adjustment and supply line adjustment terms, prevents unwanted oscillations. The effect of using the same or different adjustment times is relative. Therefore, in the literature, it is suggested that a weight coefficient should be used instead of explicitly using two separate adjustment times. This weight is simply equal to stock adjustment time divided by supply line adjustment time. When this weight is equal to zero, supply line is not considered at all, and when it is equal to one, the difference between the desired and actual supply line values has the same importance in the ordering equation as the difference between the desired and actual stock values. Therefore, this weight is named as weight of supply line. When weight of supply line is equal to one, the supply line is fully considered and the sum of the supply line and stock levels effectively reduces to a first order level that cannot oscillate. In this paper, we defined one more parameter that we call relative aggressiveness, which is equal to acquisition delay time divided by stock adjustment time. According to our experience, the existence or non-existence of stable or unstable oscillations is a function of the order of the material supply line delay structure, weight of supply line, and relative aggressiveness. Usually, acquisition delay time and order of the supply line delay structure are not decision parameters; weight of supply line and stock adjustment time are. In this paper, we are aiming to give more insight to the readers about the selection of the values for these two important parameters.

Keywords: stock management; material supply line delay; order of delay; weight of supply line; acquisition delay time; stock adjustment time; relative aggressiveness.

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1. Introduction

It is known that the presence of a material supply line delay may lead to unwanted oscillatory stock behavior (Barlas and Ozevin, 2004; Sterman, 1987, 1989; Yasarcan, 2010; Yasarcan and Barlas, 2005a). It is also well known that fully considering the supply line in the ordering decisions, which means using the same adjustment time for stock adjustment and supply line adjustment terms, prevents unwanted oscillations. The effect of using the same or different adjustment times is relative. Therefore, in the literature, it is suggested that a weight coefficient should be used instead of explicitly using two separate adjustment times. This weight is simply equal to stock adjustment time divided by supply line adjustment time. When this weight is equal to zero, supply line is not considered at all, and when it is equal to one, the difference between the desired and actual supply line values has the same importance in the ordering equation as the difference between the desired and actual stock values. Therefore, this weight is named as weight of supply line (Chapter17 in Sterman, 2000).

In this paper, we defined one more decision parameter that we call relative aggressiveness, which is equal to acquisition delay time divided by stock adjustment time. According to our experience, the existence or non-existence of stable or unstable oscillations is a function of the order of the material supply line delay structure, weight of supply line, and relative aggressiveness. In this paper, we built a stock management model, examined the effects of the order of delay, *Weight of Supply Line* and *Relative Aggressiveness* values for different cases and came up with additional suggestions to currently accepted stock management heuristics.

2. Stock Management Structure

A simple stock management structure is used for modeling purposes. The main stock is controlled via using an "Anchor and Adjust" heuristic.

Stock equations are given below:

$$Stock_0 = 9 \quad [unit] \tag{1}$$

$$Stock_{t+DT} = Stock_t + (Acquisition\ Flow - Loss\ Flow) \cdot DT \quad [unit] \tag{2}$$

$$Supply\ Line_0 = Desired\ Supply\ Line \quad [unit] \tag{3}$$

$$Supply\ Line_{t+DT} = Supply\ Line_t + (Control\ Flow - Acquisition\ Flow) \cdot DT \quad [unit] \tag{4}$$

Flow equations are as follows:

$$Acquisition\ Flow = Supply\ Line / Acquisition\ Delay\ Time \quad [unit/time] \tag{5}$$

$$Control\ Flow = Loss\ Flow + Stock\ Adjustment + \begin{pmatrix} Supply\ Line \\ Adjustment \end{pmatrix} \quad [unit/time] \tag{6}$$

$$Loss\ Flow = 2 \quad [unit\ /\ time] \tag{7}$$

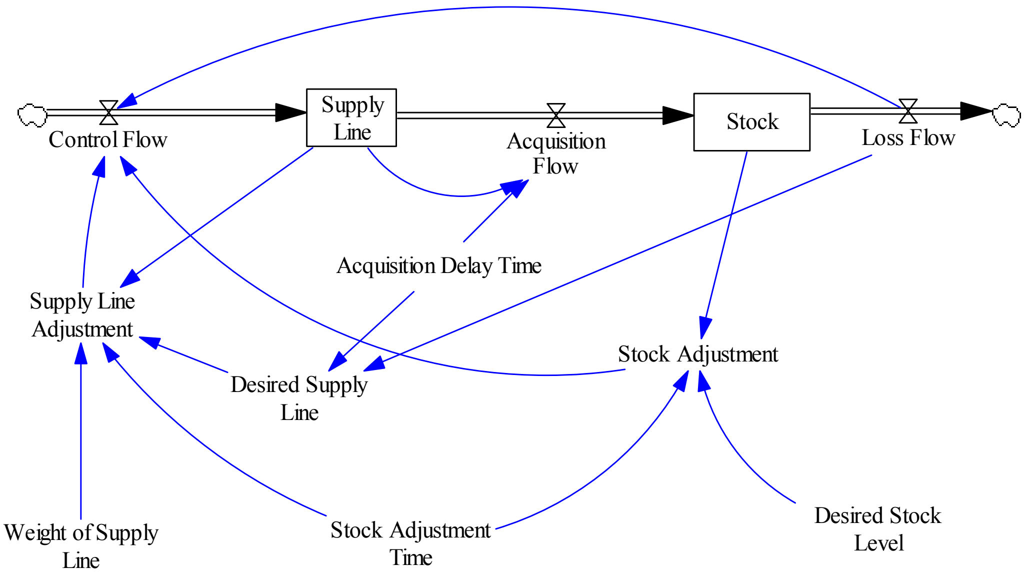

Figure 1. Stock Management Model

Model constants and the other model equations are:

$$Acquisition\ Delay\ Time = 8 \quad [time] \tag{8}$$

$$Desired\ Stock\ Level = 9 + STEP(1,1) \quad [unit] \tag{9}$$

$$Desired\ Supply\ Line = Acquisition\ Delay\ Time \cdot Loss\ Flow \quad [unit] \tag{10}$$

$$\begin{pmatrix} Supply \\ Line \\ Adjustment \end{pmatrix} = \frac{\begin{pmatrix} Weight\ of \\ Supply\ Line \end{pmatrix} \cdot (Desired\ Supply\ Line - Supply\ Line)}{Stock\ Adjustment\ Time} \quad [unit/time] \tag{11}$$

$$Stock\ Adjustment = \frac{Desired\ Stock\ Level - Stock}{Stock\ Adjustment\ Time} \quad [unit/time] \tag{12}$$

The values of *Weight of Supply Line* (*WSL*) and *Stock Adjustment Time* are not presented with other equations because they are the decision parameters of our stock management model. In other words, they determine the different ordering strategies used in the model.

Weight of Supply Line shows the importance given to *Supply Line* with respect to *Stock*. It is formulated as stock adjustment time divided by supply line adjustment time. When *Weight of Supply Line* is equal to one, *Supply Line* has the same importance level as *Stock* in determining the orders. Moreover, the sum of *Supply Line* and *Stock* levels effectively reduces to a first order level that cannot oscillate (Yasarcan and Barlas, 2005a). When it is equal to zero, the level of *Supply Line* has no importance in the ordering decisions.

In this study, the stocks start at their equilibrium levels and a shock is given to *Desired Stock Level* by 1 unit at time 1 in order to create a perturbation in the system. Therefore, *Stock* starts to seek its new desired level after time 1. The aim is to examine the performances of various set of parameters during the process of *Stock* seeking its new desired level. We defined a penalty function to compare the effect of different sets of values of *Weight of Supply Line* and *Stock Adjustment Time*. Our penalty is the cumulative of absolute value of discrepancy between *Stock* and *Desired Stock Level* up to time 1000 which is our standard simulation time.

3. Relative Aggressiveness

Relative Aggressiveness is equal to *Acquisition Delay Time* divided by *Stock Adjustment Time*. For a given order of material supply line delay and given *Weight of Supply Line* value, it is the value of *Relative Aggressiveness* that determines whether the stock behavior is oscillatory (stable or unstable), or non-oscillatory. Thus, if one increases or decreases both *Acquisition Delay Time* and *Stock Adjustment Time* with the same ratio, the characteristic of the behavior (stable oscillations, unstable oscillations, or non-oscillatory dynamics) will remain the same.

There are two critical points determined by the value of *Relative Aggressiveness*. First one is the point where *Stock* starts to make stable oscillations and second one is the point where *Stock* starts to make unstable oscillations. When the value of *Relative Aggressiveness* is between zero and the first critical point, *Stock* seeks its desired level without oscillating. When the value of *Relative Aggressiveness* is between the first and second critical points, *Stock* shows stable oscillations. And lastly, if the value of *Relative Aggressiveness* is above the second critical point, oscillations are unstable.

We tried to find critical *Relative Aggressiveness* values for three different levels (0, 0.1, and 0.5) of *Weight of Supply Line* and five different levels of delay orders (1st order, 2nd order, 3rd order, 5th order, and infinite order[2]) by using simulations. These values are presented in the following tables. However, the values we found are not the theoretical values and they include numerical errors.

Table 1. Critical *Relative Aggressiveness* values for *WSL* = 0

	DO1	DO2	DO3	DO5	Discrete
Stable	0,27	0,31	0,33	0,35	0,38
Unstable	for no value	3,72	2,55	2,02	1,56

Table 2. Critical *Relative Aggressiveness* values for *WSL* = 0.1

	DO1	DO2	DO3	DO5	Discrete
Stable	0,28	0,33	0,35	0,37	0,40
Unstable	for no value	7,70	3,65	2,60	1,87

Table 3. Critical *Relative Aggressiveness* values for *WSL* = 0.5

	DO1	DO2	DO3	DO5	Discrete
Stable	0,37	0,45	0,48	0,52	0,57
Unstable	for no value	Not known	Not known	Not known	Not known

It is known that when *Weight of Supply Line* is one, *Stock* cannot oscillate. So, we did not put the table for this case. It is also known that when the delay order is one and *Weight of Supply Line* is zero, *Stock* cannot show unstable oscillatory behavior (Yasarcan, 2003). This is also valid for any value of *Weight of Supply Line* when the delay order is one. For the case where *Weight of Supply Line* is 0.5, we were not able to calculate the points where unstable oscillations begin due to the existing level of numerical precision provided by the simulation software that we used in the experiments.

The table below shows the optimum *Relative Aggressiveness* values. Since 2nd order and 5th order delays are not used in our penalty comparisons, they are not included in this table. When we examine tables 1, 2, 3, and 4 together, we observe that the optimum *Relative Aggressiveness* values seen in Table 4 corresponds to stable oscillation phase of stocks. Note that *Weight of Supply Line* = 1 is an exception for this because *Weight of Supply Line* = 1 completely eliminates oscillations. This shows that stable oscillations are more desirable for a stock than a non-oscillatory goal-seeking behavior. This result is quite surprising because there is a strong focus in the literature trying to completely

[2] The order of a discrete delay is infinite.

eliminate all types of oscillations. One should also keep in mind that strong oscillations are undesirable and it is only slight, sometimes unnoticeably small, oscillations that are desired.

Table 4. Optimum *Relative Aggressiveness* values producing the minimum penalty values for a given delay order and *WSL* value

	WSL 0	**WSL 0.1**	**WSL 0.5**	**WSL 1**
DO1	4,32	16 +	16 +	infinity
DO3	0,73	0,85	16 +	infinity
Discrete	0,59	0,65	1,13	infinity

4. The Effect of Delay Order and Relative Aggressiveness Values on the Penalty Values for a Given Weight of Supply Line Value

The figures plotted below confirm that increasing the order of delays have a negative effect on penalties as expected. When *Relative Aggressiveness* value exceeds the point where unstable oscillations begin, the negative effect of increasing the order of delays becomes even larger and penalties start to increase excessively.

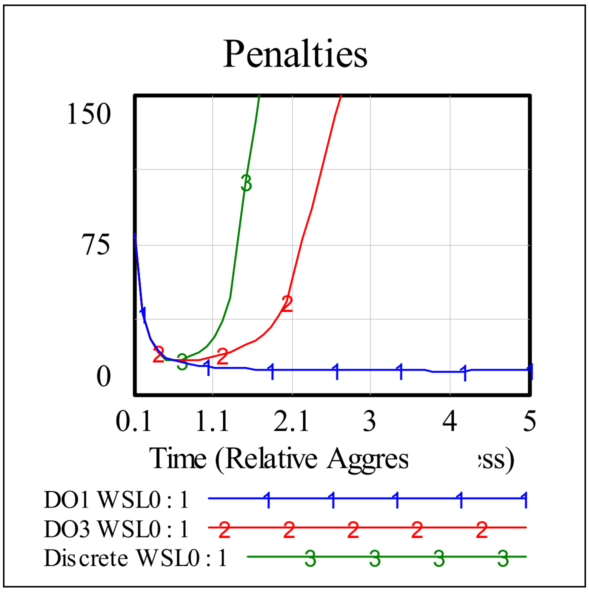

Figure 2. Penalties for *WSL* = 0

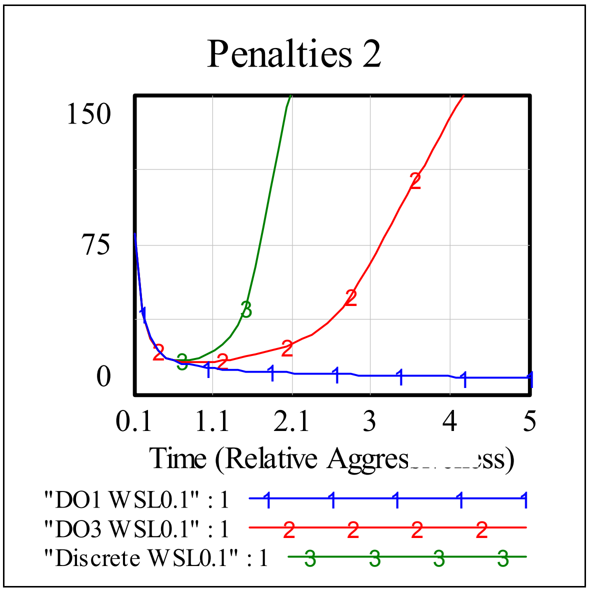

Figure 3. Penalties for *WSL* = 0.1

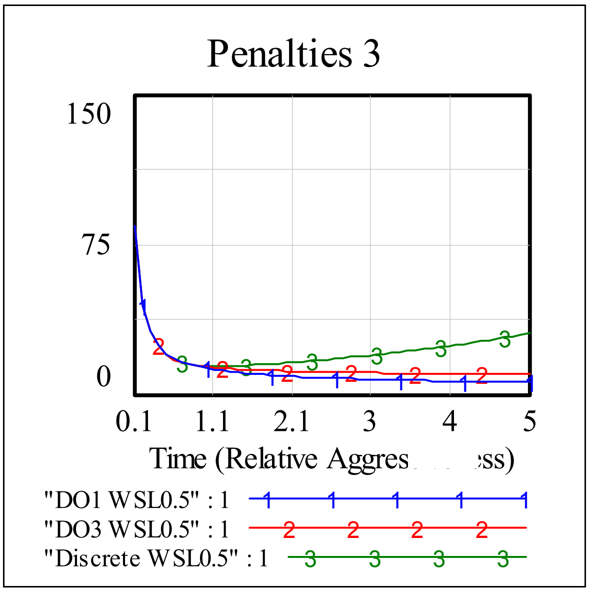

Figure 4. Penalties for *WSL* = 0.5

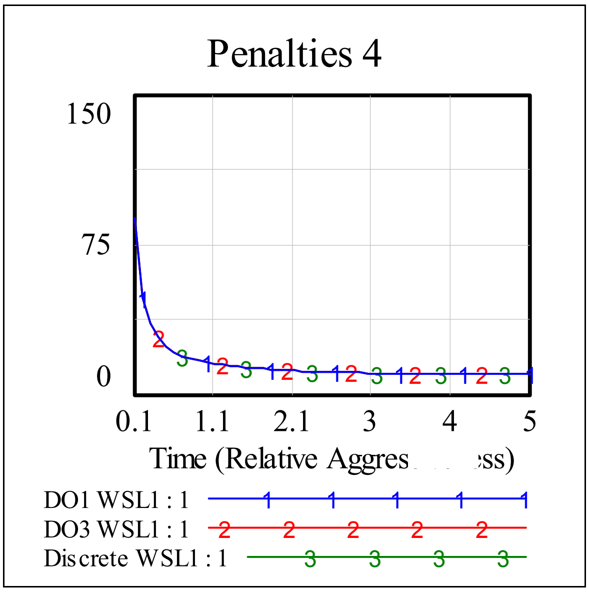

Figure 5. Penalties for *WSL* = 1

5. The Effect of Delay Order on the Stock Behavior for Given Weight of Supply Line and Relative Aggressiveness Values

We know the negative effect of increasing the order of delays from previous section. In this section, we see that for low *Weight of Supply Line* values, this negative effect is more obvious. Figure 6 and Figure 7 show that the heuristics with lower *Weight of Supply Line* values have more tendency to create oscillations than the heuristics with higher *Weight of Supply Line* values. Note that these oscillations mentioned here are stronger than the desirable minor oscillations that we mentioned at the end of Section 3.

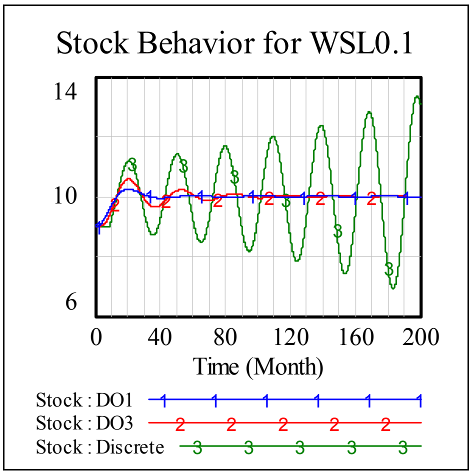

Figure 6. Stock Behavior for different delay orders (*WSL* = 0.1, *Relative Aggressiveness* = 2)

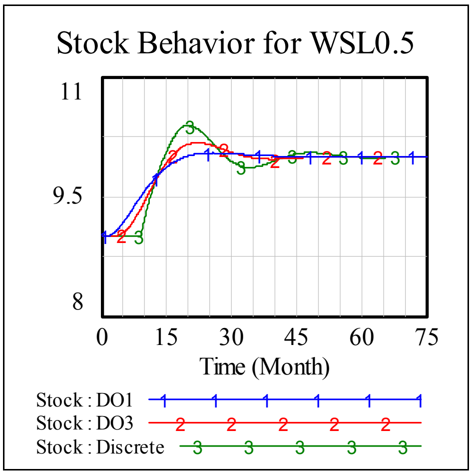

Figure 7. Stock Behavior for different delay orders (*WSL* = 0.5, *Relative Aggressiveness* = 2)

6. The Effect of Weight of Supply Line and Relative Aggressiveness Values on the Penalty Values for a Given Delay Order

Figures 8, 9, and 10 show the effect of *Weight of Supply Line* (0, 0.1, 0.5 and 1) and *Relative Aggressiveness* on the penalty values for given different delay orders. It can be seen from figures 8 and 9 that *Weight of Supply Line* = 0.5 is as stable as Weight of Supply Line =1 when delay order is low. In this experiment, *Relative Aggressiveness* values between 0.1 and 16 are used. Even when *Relative Aggressiveness* value is 16 and delay order is 3, the penalty of *Weight of Supply* = 0.5 is lower than the penalty of *Weight of Supply Line* = 1. The penalty of first one is 7.379 while the latter one is 8.5. And lastly, figure 8 agrees with tables 1, 2 and 3. When delay order is one, it is not possible to have unstable oscillations. Figure 8 also shows that the penalty values created by different parameter values for a first order delay are very close to each other. Therefore, it is easier to control a stock management system involving a first order material supply line delay compared to higher order material supply line delays.

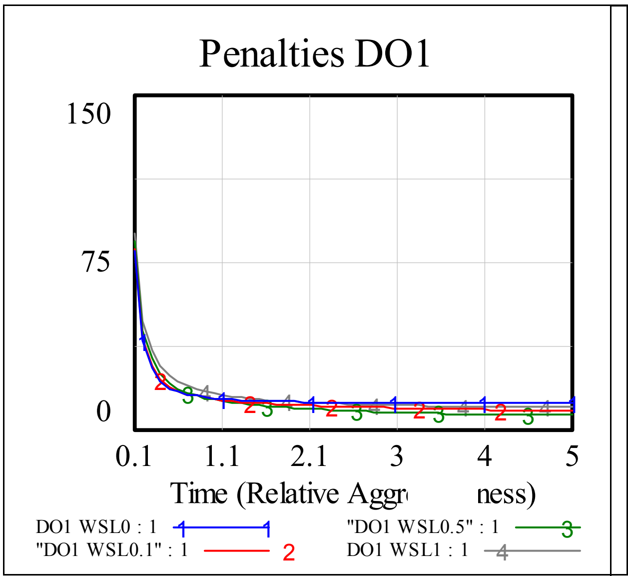

Figure 8. Penalties for delay order = 1

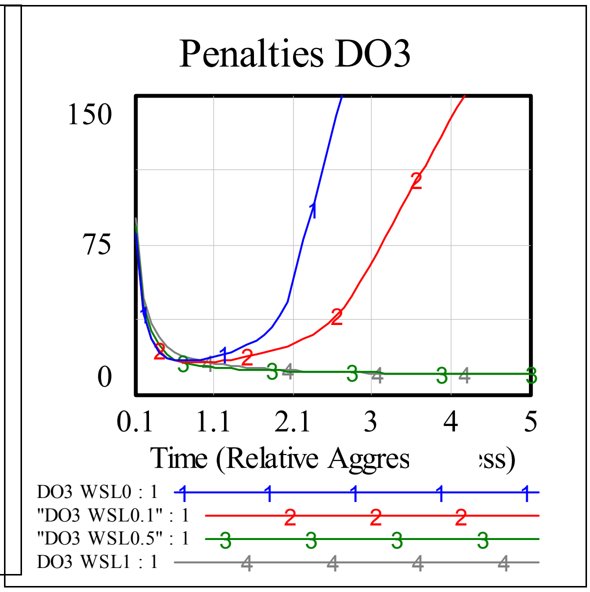

Figure 9. Penalties for delay order = 3

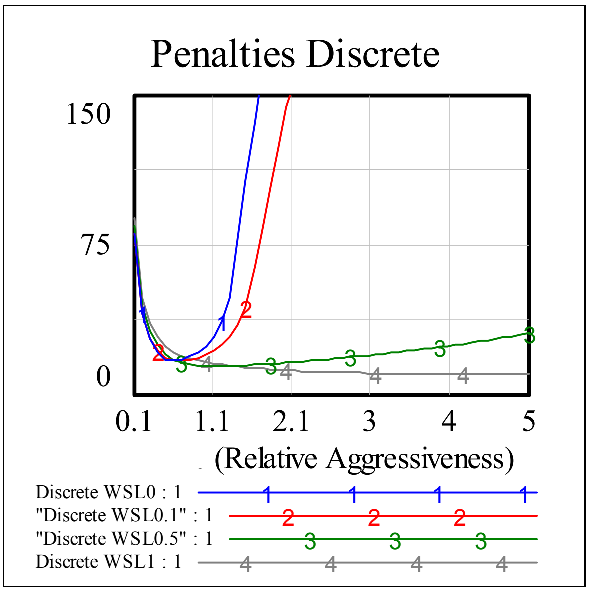

Figure 10. Penalties for delay order = ∞

7. Effect of Relative Aggressiveness when a Good Weight of Supply Line Value is Chosen

We have tried *WSL* = 0, *WSL* = 0.1, *WSL* = 0.5, and *WSL* = 1 and selected the *Weight of Supply Line* value that gives relatively the best result for 1^{st} order, 3^{rd} order, and discrete order delays. Roughly speaking, setting *Weight of Supply Line* equal to 0.5 for 1^{st} and 3^{rd} order delays and equal to 1 for discrete order delay produces the lowest penalties. The penalties are plotted for these three cases in the following figures.

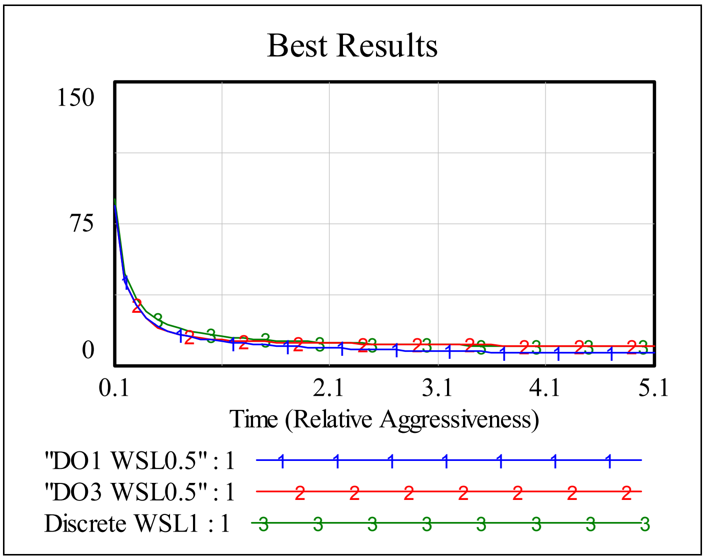

Figure 11. Penalties for good *WSL* values

It is observed that when *Weight of Supply Line* is relatively good for a given delay order, increasing *Relative Aggressiveness* yields lower penalty values. Another observation is that improving *Relative Aggressiveness* more than the value of 4 does not have a meaningful effect on penalties. Stock behaviors are given below when *Relative Aggressiveness* is 4 and *WSL* = 0.5 for first and third order supply line delays, and *WSL* = 1 for discrete (infinite order) supply line delay.

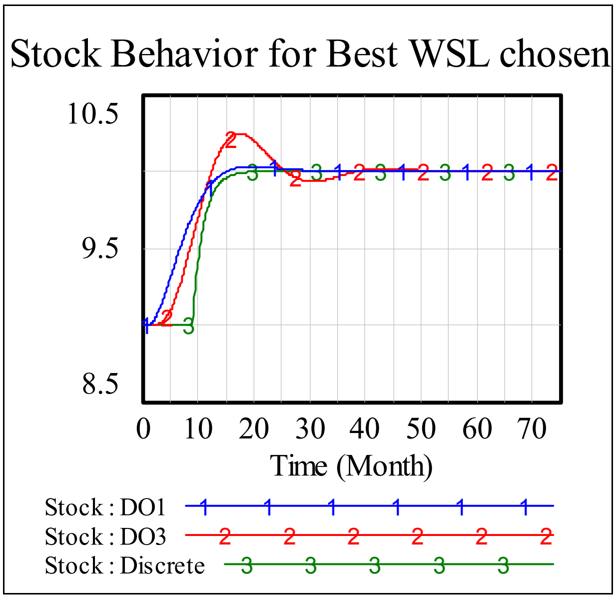

Figure 12. Stock behavior for different delay orders,
(*Relative Aggressiveness* = 4, *WSL* =0.5, 0.5, and 1 consecutively for DO1, DO3, and Discrete Delay)

8. Conclusion

In a stock management structure, the behavior of the main stock depends on *Weight of Supply Line*, order of the material supply line delay structure, *Acquisition Delay Time*, and *Stock Adjustment Time*. Usually, *Acquisition Delay Time* and the order of the material supply line delay structure are not decision making parameters; *Weight of Supply Line* and *Stock Adjustment Time* are. We observed that, the ratio of *Acquisition Delay Time* and *Stock Adjustment Time* determine the characteristics of the stock behavior (e.g. if the stock is oscillating unstably, changing the *Acquisition Delay Time* and *Stock Adjustment Time* values while keeping their ratio constant, will never make the stock stop oscillating unstably). Therefore, we defined a new parameter that we named *Relative Aggressiveness*, which is equal to *Acquisition Delay Time* divided by *Stock Adjustment Time*. *Relative Aggressiveness* can replace the two parameters (*Acquisition Delay Time* and *Stock Adjustment Time*) simplifying the analysis. Once a good *Relative*

Aggressiveness is determined, the value of *Stock Adjustment Time* can be determined by setting its value equal to *Acquisition Delay Time* divided by *Relative Aggressiveness*.

Theoretically speaking, taking *Weight of Supply Line* as one and decreasing *Stock Adjustment Time* down to zero will give the best stock behavior because setting *Weight of Supply Line* equal to one guarantees the stability of the stock and decreasing *Stock Adjustment Time* to very low values ensures faster response in the stock yielding lower penalty values. However, *Stock Adjustment Time* would normally has a lower limit in practice (Yasarcan and Barlas, 2005b). It would not be reasonable to decrease *Stock Adjustment Time* below the ordering period as it will create instability problems. Hence, taking *Weight of Supply Line* equal to one would not always give the best result in stock management.

The simulation experiments that we carried out and reported in the previous sections suggest that setting *Relative Aggressiveness* higher than 4 will not create significant improvements. Hence, setting *Stock Adjustment Time* to one fourth of *Acquisition Delay Time* will give a close optimum result. *Weight of Supply Line* value can be selected based on the order of the material supply line delay structure. Roughly speaking, for discrete delays, *Weight of Supply Line* equal to 1 will give very good results and, for continuous delays (such as first and third order), *Weight of Supply Line* equal to 0.5 will give very good results.

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