A Soft Landing Model and a Mass Spring Damper Based Control Heuristic[1]

Togay Tanyolaç and Hakan Yasarcan
Industrial Engineering Department
Bogazici University
Bebek – Istanbul 34342 – Turkey
caloynat@gmail.com; hakan.yasarcan@boun.edu.tr

Abstract
This paper presents a soft landing model and a related control heuristic. The aim of the modeling effort is to transparently represent the process of landing a spacecraft on the surface of a celestial body. The process of landing is a challenging task because there are two main contradictory performance criteria to be met simultaneously; the landing duration should be as short as possible, but at the same time crashing the spacecraft to the surface should be avoided. As an answer to this challenge, we adapted a control heuristic from the mass spring damper model using the similarity of the equations of the model presented in this paper to the equations of the mass spring damper model; both models can be reduced to a second order linear differential equation. According to the initial simulation runs, the adapted heuristic can reasonably land the spacecraft.

Keywords: soft landing; spacecraft; control heuristic; mass spring damper.

1. Introduction

Soft landing is an interesting and challenging problem in space exploration. The process of landing is a challenging task because there are two main contradictory performance criteria to be met simultaneously; the landing duration should be as short as possible, but at the same time crashing the spacecraft to the surface should be avoided. In order to achieve a fast and safe landing on the surface of a celestial body, the landing process should be controlled. When landing on celestial bodies with no atmosphere (e.g. the moon), deceleration strategies that rely on the drag force (e.g. a parachute) do not work due to the absence of atmospheric molecules. Therefore, a reverse force thruster, which will decelerate the vehicle, is needed (see Figure 1). At the instant of landing, an

[1] *This research is supported by a Marie Curie International Reintegration Grant within the 7th European Community Framework Programme (grant agreement number: PIRG07-GA-2010-268272) and also by Bogazici University Research Fund (grant no: 5025).*

impact force is generated depending on the mass, velocity, and the landing gear specifications of the spacecraft. For a successful landing, this impact force must be under a certain limit and, ideally, it should be as low as possible so as not to harm the vehicle. We assumed a constant mass and fixed specifications for the landing gear. Thus, the magnitude of the impact force can only be controlled via controlling the velocity, which should be within certain limits to prevent a crash. If the only criterion was to prevent crashing the spacecraft, that would not be difficult to achieve by slowing down the landing process. However, long landing duration necessitates extensive use of fuel, which should also be avoided. Therefore, another goal in landing is to decrease the time to land. Consequently, a reasonable landing occurs when the vehicle descends to the surface quickly, but decelerates safely to low velocity values before the instant of landing (Liu, Duan, and Teo, 2008; Zhou et al., 2009).

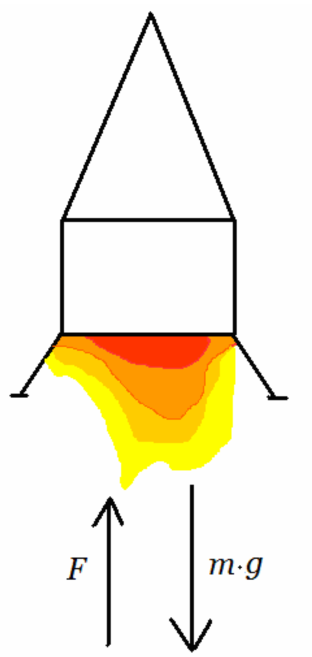

Figure 1: Free body diagram of the vehicle with a control force (F) generated by the reverse force thruster and the gravitational force ($m \cdot g$)

We modeled the soft landing challenge using System Dynamics (SD) methodology (Barlas, 2002; Forrester, 1961 and 1971; Sterman, 2000). SD has a strong focus on the correct representation of the problem related elements of the system, which increases the validity of the constructed simulation model. The aim of the modeling effort was to transparently represent the process of landing a spacecraft on the surface of a celestial body. SD methodology serves the purpose of explicit representation of the model variables and parameters, which facilitates sharing and understanding of the model structure and dynamics (Barlas, 2002; Forrester, 1961 and 1971; Sterman, 2000).

As described before, the main goal in the soft landing problem is to land the spacecraft as gently and as fast as possible. In the fourth section, we present a control heuristic adapted from the mass spring damper model that guarantees safe landing conditions for the spacecraft according to the initial simulation runs that we obtained (third section). The total duration of landing seems plausible, as well.

2. The Model Structure and Equations

In this study, we first constructed a stock-flow model of the soft-landing problem, which is given in Figure 2. This diagram represents only the physical structure of the problem described in the previous section; it does not represent the controller (e.g. a human decision maker, a computer). *Height* and *Velocity* are the two stock variables in the model. *Velocity*, which is a stock variable, is at the same time the one and only flow of *Height*. *Velocity* has a single flow too; *Acceleration*. *Height* is controlled via *Velocity*, *Velocity* via *Acceleration*, *Acceleration* via *Net Force*, and *Net Force* via *Control Force* (equations 1-7)[2]. The control feedback loop also includes the controller, which determines *Control Force* of the reverse force thruster via *Desired Control Force*. Note that the natural inputs to the controller are *Height* and *Velocity*. *Control Force* cannot be more than the maximum force applicable by the thruster (equations 8 and 9).

$$Height_0 = 1000 \quad [m] \tag{1}$$

$$Height_{t+DT} = Height_t + Velocity_t \cdot DT \quad [m] \tag{2}$$

$$Velocity_0 = -10 \quad [m/s] \tag{3}$$

$$Velocity_{t+DT} = Velocity_t + Acceleration \cdot DT \quad [m/s] \tag{4}$$

$$Acceleration = Net\ Force / Mass \quad [m/s^2] \tag{5}$$

$$Mass = 1000\ [kg] \tag{6}$$

$$Net\ Force = Gravitational\ Force + Damping\ Force + Control\ Force \quad [N] \tag{7}$$

$$Control\ Force = \begin{Bmatrix} 0, & Height \le 0 \\ Desired\ Control\ Force, & \begin{cases} Height > 0, \\ Max\ Force \le Desired\ Control\ Force \end{cases} \\ Max\ Force, & \text{otherwise} \end{Bmatrix} \quad [N] \tag{8}$$

[2] One Newton amounts to the force needed to increase the velocity of a one kilogram body of mass by one meter per second in one second ($N = kg \cdot m / s^2$).

$$Max\ Force = 30{,}000\,[N] \tag{9}$$

Positive *Height*, *Velocity*, *Acceleration*, and force directions are upward from the surface. *Height* equals zero means that the vehicle touches the ground, but the springs of the landing gear are at rest, so they bear no force at *Height* equals zero. Thus, when the vehicle comes to a static equilibrium, the springs of the landing gear get compressed balancing the weight (*Gravitational Force*) of the vehicle and *Height* becomes slightly less than zero.

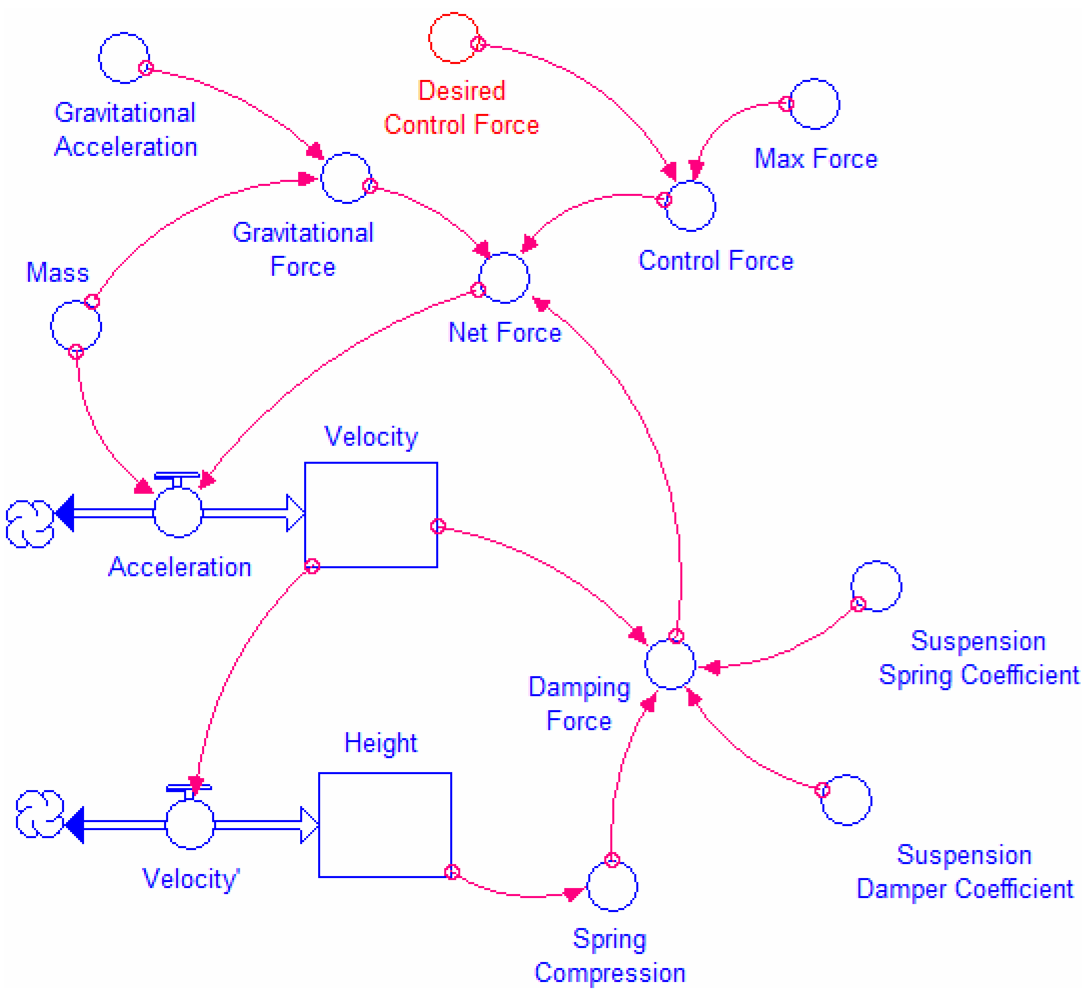

Figure 2: Stock-flow diagram of the model

Gravitational Force, *Damping Force*, and *Control Force* add up to the *Net Force* acting on the vehicle (Equation 7). *Gravitational Force* acts on the vehicle due to mass and gravity (Equation 10). Corollary to constant *Mass* (Equation 6) and constant *Gravitational Acceleration* (Equation 11) assumptions, *Gravitational Force* is also a constant (Equation 10). The gravitational acceleration of the celestial body to be landed on is assumed to be equal to the surface gravitational acceleration of Venus that is 8.87 m/s^2 (Equation 11). Note that the assumed landing conditions other than the gravitational acceleration do not resemble the conditions of Venus at all.

$$\textit{Gravitational Force} = \textit{Mass} \cdot \textit{Gravitational Acceleration}\ [N] \tag{10}$$

$$\textit{Gravitational Acceleration} = 8.87\ [m/s^2] \tag{11}$$

The landing gear of the spacecraft is comprised of dampers and springs. *Damping Force*, which is a result of the compression of the landing gear, is generated after the spacecraft contacts the landing surface (Equation 12). To be able to correctly represent the conditional existence of *Damping Force*, we also defined a variable named *Spring Compression*, which represents the amount of compression of the landing gear (Equation 13). Inclusion of *Spring Compression* is in accordance with our aim of obtaining a transparent model. *Suspension Spring Coefficient*, *Suspension Damper Coefficient*, and *Mass* determine the damping behavior, which can be subcritical, critical, or supercritical. The values of the two coefficients are selected such that a critically damped behavior is obtained after the touchdown[3].

$$\textit{Damping Force} = \left\{\begin{matrix} 0, & \textit{Spring Compression} = 0 \\ \begin{pmatrix}\textit{Suspension}\\ \textit{Spring}\\ \textit{Coefficient}\end{pmatrix} \cdot \begin{pmatrix}\textit{Spring}\\ \textit{Compression}\end{pmatrix} - \begin{pmatrix}\textit{Suspension}\\ \textit{Damper}\\ \textit{Coefficient}\end{pmatrix} \cdot \textit{Velocity}, & \text{otherwise}\end{matrix}\right\} [N] \tag{12}$$

$$\textit{Spring Compression} = \begin{pmatrix} 0, & \textit{Height} \geq 0 \\ -\textit{Height}, & \text{otherwise}\end{pmatrix}\ [m] \tag{13}$$

The simplifying model assumptions are given below:

- The movement of the spacecraft in the horizontal axes is not modeled. Spacecraft is assumed to move only vertically.
- There is no atmosphere in the landing area, thus no air friction exists that would

[3] $\textit{Suspension Spring Coefficient} = 17{,}740\ [N/m]$

$\textit{Suspension Damping Factor} = 2\ [\text{dimensionless}]$

$\textit{Suspension Damper Coefficient} = \begin{pmatrix}\textit{Suspension}\\ \textit{Damping Factor}\end{pmatrix} \cdot \sqrt{\begin{pmatrix}\textit{Suspension}\\ \textit{Spring Coefficient}\end{pmatrix} \cdot \textit{Mass}}\ \left[\frac{N \cdot s}{m}\right]$

cause a drag force on the vehicle.

- *Gravitational Acceleration* is assumed to be constant during landing, it does not change with the distance to the surface.
- *Mass* is a constant, the change in the mass due to fuel consumption is ignored.
- There are no delays caused by actuators; *Desired Control Force* generated by the controller affects *Control Force* without a time lag.
- Information flow from the system to the controller is perfect and instantaneous; There are no errors or delays caused by measurement processes.
- Upon touching the ground, the thruster is off and is not switched on again. The simplified model diagram in Figure 2 and Equation 8 do not reflect this assumption. By giving the simplified version of the model, we aim to improve the readability of the manuscript and prevent digression.

3. Dynamic Behavior of Landing

As described in the previous section, *Height* is controlled via *Velocity* (Equation 2), *Velocity* via *Acceleration* (Equation 4), *Acceleration* via *Net Force* (Equation 5), and *Net Force* via *Control Force* (Equation 7). The control feedback loop also includes the controller, which determines *Control Force* applied by the reverse force thruster via *Desired Control Force*. In order to obtain a reasonable value for *Desired Control Force*, the controller should consider the system state variables (i.e. *Height* and *Velocity*). Only by doing so is it possible to reach the aim of landing the spacecraft as gently and as fast as possible. Even under the simplifying assumptions listed in the previous section, the control task remains a challenging one because it is quite difficult to appropriately consider the system state information in the decisions. The main reason for the difficulty is that the control task requires simultaneous control of *Height* and *Velocity*, which –due to the physical structure of the problem– can only be indirectly affected by the reverse force thruster; *Height* and *Velocity* have inertia; their values do not change instantaneously (see Figures 1 and 2 and equations 1-7).

The stock-flow model given in Figure 2 and defined by equations 1-9 describes the structure of the soft landing problem excluding the controller. The formulations of the heuristic suggested for the controller is explained in the next section. The dynamic behavior presented in figures 3-7 is generated by simulating the model including the controller with the proposed heuristic for 120 seconds (equations 1-13 and equations 17-18).

The dynamic behavior of *Height* is given in Figure 3. Initially, the change in *Height* (i.e. *Velocity*) is relatively fast and, as the spacecraft approaches to the surface, the change in *Height* slows down. Hence, the behavior obtained by the control heuristic is a reasonable one; by a fast initial decline, the heuristic tries to decrease the time to land; by a slow final approach, it keeps the impact force well below harmful values. At the instant of touchdown, the value of *Velocity* is -0.05 meters per second (-0.18 km/h) creating a maximum impact force of circa 10,090 Newton, approximately 1.14 times the weight of the spacecraft on the target celestial body (8,870 Newton). The weight corresponds to the model variable *Gravitational Force*, which is the force that the landing gear must bear when the spacecraft is standing still on the ground.

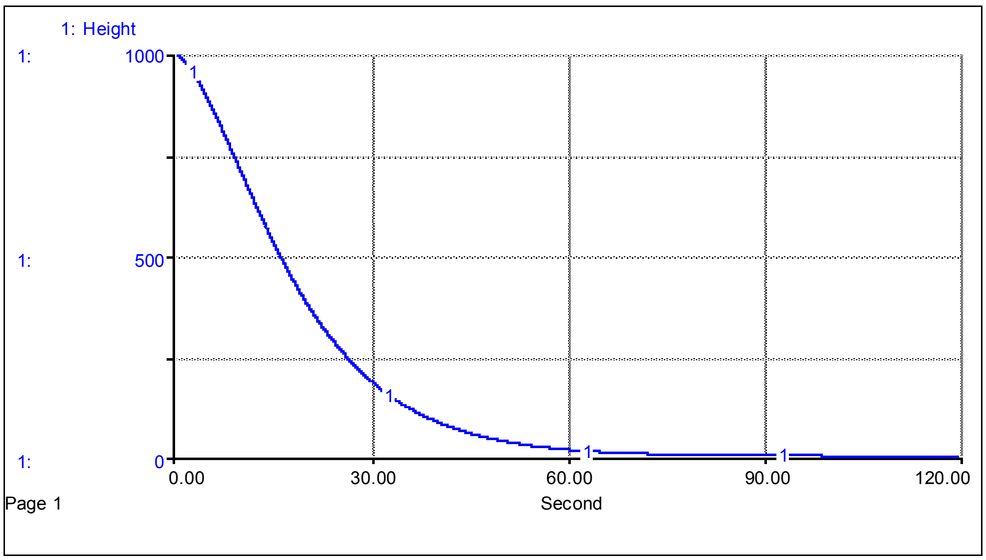

Figure 3: Dynamic behavior of *Height*

The dynamic behavior of *Velocity* and *Net Force* acting on the vehicle during landing are given in figures 4 and 5, which further explain the dynamic behavior obtained by the control heuristic. At first, the heuristic allows the spacecraft to accelerate in the negative direction towards the landing surface (see Figure 4, approximately within the time range of 0-10 seconds) by keeping *Net Force* negative (i.e. *Control Force* less than *Gravitational Force,* see figures 5 and 6). Aiming to decrease the duration of landing, *Velocity* continues to increase during this initial period. After this initial phase, *Velocity* decreases until the vehicle touches the surface (see Figure 4, approximately within the time range of 10-100 seconds). In this later phase, the heuristic produces more *Control Force* than *Gravitational Force* (Figure 6) resulting in a positive *Net Force* (Figure 5). At the moment of landing, *Control Force* is turned off and *Damping Force*, which is zero throughout the simulation up to this point, takes over and stops the vehicle (see figures 5 and 6, approximately around 100 seconds).

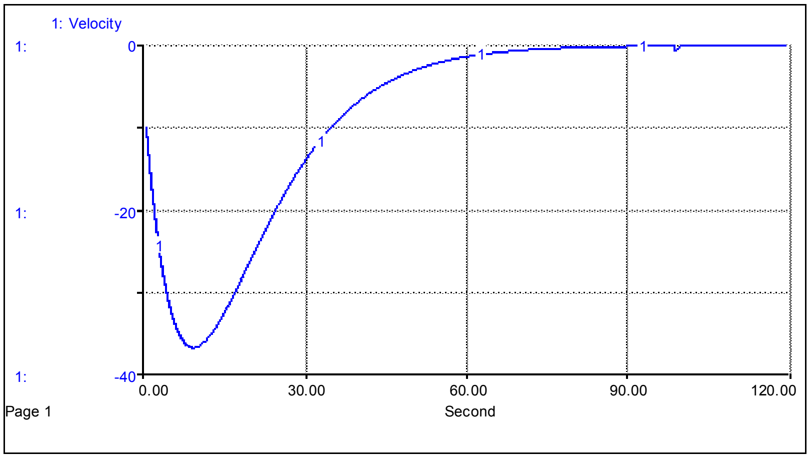

Figure 4: Dynamic behavior of *Velocity*

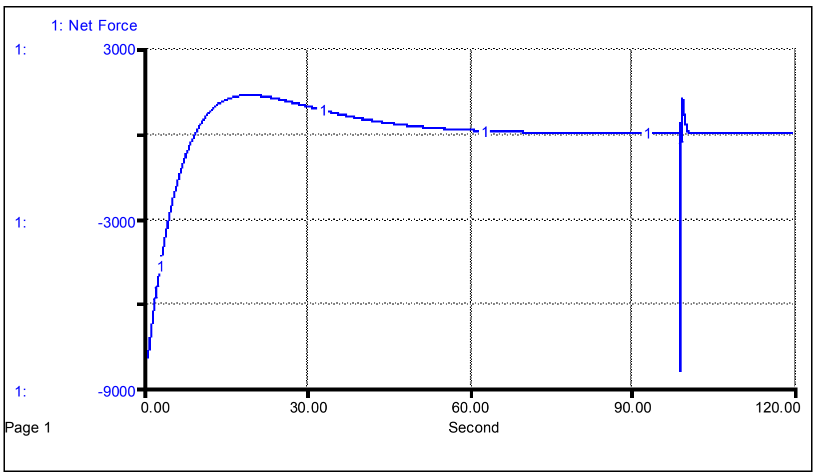

Figure 5: Net force acting on the vehicle during landing

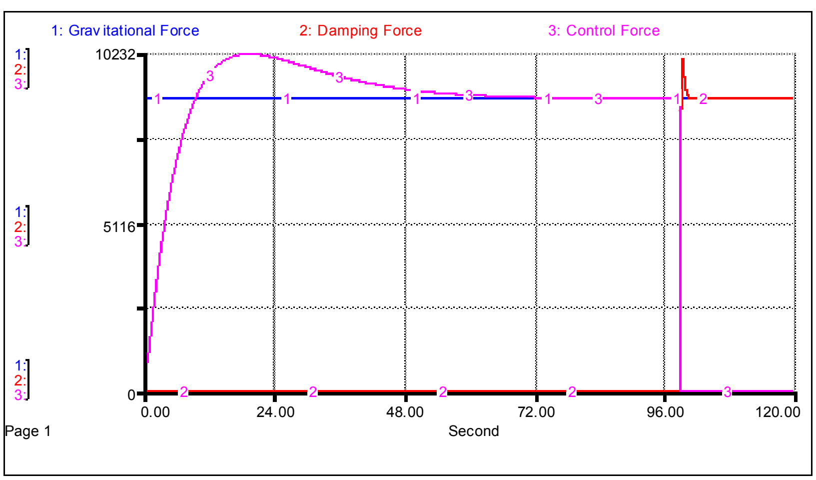

Figure 6: Absolute values of the forces acting on the vehicle during landing[4]

[4] In order to ease the comparison of the different forces acting on the vehicle, the directions of the forces are ignored on this diagram.

4. A Mass spring Damper Based Control Heuristic

The stock-flow model given in Figure 2 represents only the physical structure of the soft landing problem. However, the simulated behavior discussed in the previous section is generated by the model including the suggested mass spring damper based control heuristic, which is assumed to be used by the controller in producing the values for *Desired Control Force*. The aim of this section is to present the formulations of this heuristic.

Yasarcan and Barlas (2005) uses a procedure in developing control heuristics for control problems involving information delay or indirect control via a secondary-stock. This procedure adapts a well known successful heuristic for control problems involving material supply line delay, using the similarity of the differential equations of control problems involving different types of delay structures. The model presented in this paper can be reduced to a second order linear differential equation because it contains two stock variables, which are defined by approximate integral equations (Equation 2 and Equation 4). The mass spring damper model is well studied and it is known how to obtain a certain behavior by adjusting the model parameter values. Furthermore, it can also be represented by a second order linear differential equation. The heuristic suggested in this paper is developed based on the similarity of the differential equations of the mass spring damper model and the model presented in this paper[5].

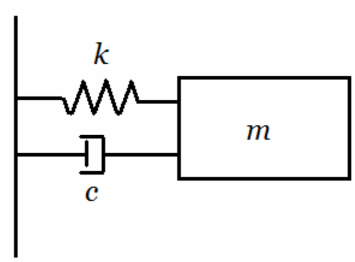

Figure 7: Mass spring damper schematic

The schematic given in Figure 7 is a well known one. The differential equation of a non-driven (i.e. $F_{external} = 0$) mass spring damper model with mass m, spring constant k, and damper coefficient c is given below:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = 0 \tag{14}$$

[5] The authors of this paper acknowledge that it is Dr. I. Emre Köse who suggested to us to use the mass spring damper model for this purpose.

In Equation 14, x represents displacement, $\dot{x}$ represents velocity, and $\ddot{x}$ represents acceleration. This equation can be described by using stock-flow concepts, x and $\dot{x}$ being the stocks and their associated flows being $\dot{x}$ and $\ddot{x}$ respectively. Note that $\dot{x}$ is a flow and a stock at the same time. As a further clarification, $-k \cdot x$ is the spring force (F_{spring}) and $-c \cdot \dot{x}$ is the damper force (F_{damper}). The net force applied on the body of mass is the sum of these two forces ($F_{net} = F_{damper} + F_{spring} = -c \cdot \dot{x} - k \cdot x$). According to Newton's second law of motion mass times acceleration is equal the net force acting on the body ($F_{net} = m \cdot \ddot{x}$). Therefore, mass times acceleration is equal to the sum of the spring force and damper force. Hence, Equation 14 is obtained.

The damping ratio ζ of the mass spring damper model defined by Equation 14 is:

$$\zeta = \frac{c}{2 \cdot \sqrt{m \cdot k}} \tag{15}$$

The dynamics of the mass spring damper model can be underdamped, overdamped, or critically damped depending on the value of the damping ratio ζ. For ζ values under 1, the dynamic behavior is underdamped and for ζ values over 1, it is overdamped. The case where the damping ratio ζ is exactly 1 is called critically damped. When the dynamic behavior is underdamped, the spring dominates the movement and the body oscillates. In the critically damped case, the body asymptotically approaches the rest condition without an overshoot. In the overdamped case, the damper dominates the dynamics and the body approaches the rest condition slower compared to the critically damped case (Åström and Murray, 2008). As a summary, the importance of ζ is that determining its value determines the dynamics of the mass spring damper model.

The suggested control heuristic is adapted from the mass spring damper model that is defined by Equation 14. *Height*, *Velocity*, *Acceleration*, and *Mass* in our model corresponds to x, $\dot{x}$, $\ddot{x}$, and m in Equation 14, respectively. In the heuristic, we named k as *Height Coefficient* and c as *Velocity Coefficient*. Thus, Equation 14 becomes:

$$Mass \cdot Acceleration + \begin{pmatrix} Velocity \\ Coefficient \end{pmatrix} \cdot Velocity + \begin{pmatrix} Height \\ Coefficient \end{pmatrix} \cdot Height = 0 \tag{16}$$

Utilizing Newton's second law of motion, the following can be written:

$$\textit{Desired Net Force} = -\begin{pmatrix} \textit{Velocity} \\ \textit{Coefficient} \end{pmatrix} \cdot \textit{Velocity} - \begin{pmatrix} \textit{Height} \\ \textit{Coefficient} \end{pmatrix} \cdot \textit{Height} \quad [N] \tag{17}$$

The reverse force thruster should also counteract *Gravitational Force*. Hence, *Desired Control Force*, which is the output of the heuristic and an input to *Control Force* (see Equation 8 and Figure 2), can be given as:

$$\textit{Desired Control Force} = \textit{Desired Net Force} + \textit{Gravitational Force} \quad [N] \tag{18}$$

The parameters of the adapted heuristic, *Height Coefficient* and *Velocity Coefficient* values are set to 10 $[N/m]$ and 200 $[N \cdot s/m]$, respectively. Consequently, the damping ratio ζ for our model becomes:

$$\zeta = \frac{\textit{Velocity Coefficient}}{2 \cdot \sqrt{\textit{Mass} \cdot \textit{Height Coefficient}}} = \frac{200}{2 \cdot \sqrt{1000 \cdot 10}} = 1 \tag{19}$$

The value of the damping ratio means that the suggested control heuristic produces a critically damped behavior for the height of the spacecraft.

5. Conclusions and Future Research

In this study, we first developed a soft landing model using System Dynamics methodology. The modeling effort was focused on obtaining a valid and transparent representation of the soft landing challenge, which is to land the spacecraft as gently and as fast as possible. The main reason for the challenge is that the control task requires simultaneous control of the height and velocity of the spacecraft, which have inertia and can only be indirectly affected by the reverse force thruster. We also presented a control heuristic, which is adapted from the mass spring damper model, as an answer to this challenge. According to the initial simulation runs that we obtained, the control heuristic guarantees safe landing conditions for the spacecraft. Also, the total duration of landing is reasonably short.

The simulation model presented in this paper can be used to introduce dynamic complexity to physics and engineering students or as an introductory learning tool for the control of physical systems, and also as a platform for simulation experiments (simulation-based discovery learning environment). In the continuation of this study, we plan to extend our model by adding an action formation delay, which is assumed to be caused by an actuator, and a measurement/report formation delay, which is assumed to be caused by a sensor. The addition of these delays to our model will make it more realistic because the actuators and sensors present in a soft landing system contribute to the dynamic complexity of that system as they are sources of delays. We anticipate that the addition of these delays will cause deterioration in the dynamic behavior to a great extent. In order to overcome the problematic behavior, we plan to adapt and use the heuristics developed by Yasarcan and Barlas (2005) and Yasarcan (2011), which are specifically suitable for this kind of control problems. It is also possible to develop a soft landing game based on the model as a platform for learning and dynamic decision making experimentation.

References

Åström KJ, Murray RM. 2008. *Feedback Systems: An Introduction for Scientists and Engineers.* Princeton University Press.

Barlas Y. 2002. System Dynamics: Systemic Feedback Modeling for Policy Analysis. *Knowledge for Sustainable Development - An Insight into the Encyclopedia of Life Support System.* UNESCO/EOLSS Publishers: Paris, France; Oxford, UK.

Forrester JW. 1961. *Industrial Dynamics*. Pegasus Communications: Waltham, MA.

Forrester JW. 1971. *Principles of Systems*. Pegasus Communications: Waltham, MA.

Liu XL, Duan GR, Teo KL. 2008. Optimal soft landing control for moon lander. *Automatica*. **44**(4): 1097–1103.

Sterman JD. 2000. *Business Dynamics: Systems Thinking and Modeling for a Complex World*. Irwin/McGraw-Hill; Boston, MA.

Yasarcan H. 2011. Stock Management in the Presence of Significant Measurement Delays. *System Dynamics Review*. **27**(1): 91–109.

Yasarcan H, Barlas Y. 2005. A Generalized Stock Control Formulation for Stock Management Problems Involving Composite Delays and Secondary Stocks. *System Dynamics Review.* **21**(1): 33–68.

Zhou J, Zhou D, Teo KL, Zhao G. 2009. Nonlinear Optimal Feedback Control for Lunar Module Soft Landing, in IEEE (ed). *IEEE International Conference on Automation and Logistics*. Shenyang, China: IEEE.